

4/PHY-252 Syllabus-2023

2026

(May-June)

FYUP : 4th Semester Examination

PHYSICS

(Quantum Mechanics—I)

(PHY-252)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions.

Answer any **ten** questions.

1. (a) Derive the relation between momentum of a photon and its wavelength. 3
- (b) Calculate the Compton shift in Compton scattering experiment. $4\frac{1}{2}$
2. Find the group velocity and phase velocity associated with a (a) relativistic and (b) non-relativistic free particle in motion and hence, justify that a particle in motion should be represented by a wave packet instead of a single wave. $3+2\frac{1}{2}+2=7\frac{1}{2}$

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(Turn Over)

3. Using Heisenberg's uncertainty principle, calculate the binding energy of the electron in the ground state of hydrogen atom. $7\frac{1}{2}$

4. Establish Schrödinger time-dependent equation and hence find the time-independent equation for a particle having energy E . $4+3\frac{1}{2}=7\frac{1}{2}$

5. (a) Write three characteristics of Schrödinger time-dependent equation which distinguish it from classical wave equation. 3

(b) Why is it necessary to normalize a wave function? A free particle is represented by a wavefunction

$$\psi(x, 0) = e^{-\frac{x^2}{2a^2} + ikx}$$

Normalize the wavefunction. $1\frac{1}{2}+3=4\frac{1}{2}$

6. Establish the continuity equation of quantum mechanics. Show that probability current density is a real quantity. $4\frac{1}{2}+3=7\frac{1}{2}$

7. Wave function associated with a particle of mass m trapped in an infinite depth potential well-defined by $V(x)=0$, $0 \leq x \leq L$ and $V(x)=\infty$ elsewhere is given by

$$\psi(x) = A \sin \frac{n\pi}{L} x$$

Normalize the wavefunction. Also find the expectation value of position and momentum in the ground state. $1\frac{1}{2}+3+3=7\frac{1}{2}$

8. (a) Write down the Schrödinger equation for a one-dimensional (linear) harmonic oscillator. Write the quantized energy expression of the oscillator. How is this energy different from classical oscillator? $1+1+1\frac{1}{2}=3\frac{1}{2}$

(b) What are linear vector space and Hilbert space? $2+2=4$

9. (a) Explain the terms 'eigenvalue', 'eigenvector' and 'eigenvalue equation'. Show that eigenvectors correspond to different eigenvalues of an operator are orthogonal to each other. $3+2\frac{1}{2}=5\frac{1}{2}$

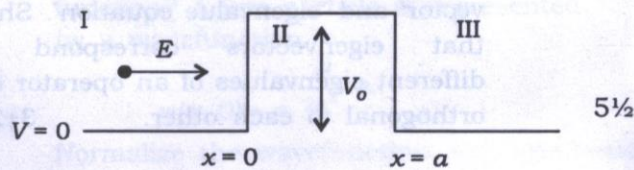
(b) What are Bra and Ket notations in matrix mechanics? What are their counterparts in wave mechanics? $1+1=2$

10. (a) What do you mean by commutation relation? What is its physical significance? Find the value of the commutation relation $[\hat{x}, \hat{p}_x]$. $1+2+2\frac{1}{2}=5\frac{1}{2}$

(b) Show that for the operators A , B and C , $[AB, C] = A[B, C] + [A, C]B$. 2

11. (a) What do you understand by expectation value of a physical observable? Write the expression for expectation value of a physical observable. $1+1=2$

- (b) A particle of mass m and energy E is incident on a potential barrier of height $V_0 (> E)$ from left (see the figure given below). Write the Schrödinger equations for the three regions. Also write the solutions. Derive the expression for transmission coefficient.



12. (a) Find the expressions for the components of angular momentum operator in cartesian coordinates. 3

- (b) Find the values of (i) $[L_x, L_y]$ and (ii) $[L^2, L_x]$. $2+2\frac{1}{2}=4\frac{1}{2}$

13. Using operator method, find the eigenvalue of L^2 . $7\frac{1}{2}$

14. How are Pauli's spin matrices related to the spin operators? Pauli's spin matrices σ_x , σ_y and σ_z , show that—

(a) $\sigma_x \sigma_y - \sigma_y \sigma_x = 2i\sigma_z$

(b) $\sigma_x \sigma_y + \sigma_y \sigma_x = 0$

(c) $\sigma_x \sigma_y = i\sigma_z$

$$1\frac{1}{2} + 3 + 2 + 1 = 7\frac{1}{2}$$

15. Find the elements of Pauli's spin matrices and hence show that $\sigma^2 = 3$. $6 + 1\frac{1}{2} = 7\frac{1}{2}$
