

6/MTH-351 Syllabus-2023

2026

(May-June)

FYUP : 6th Semester Examination

MATHEMATICS

(Dynamics—II)

(MTH-351)

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks
for the questions.*

Answer four questions, selecting one from each Unit.

UNIT—I

1. (a) A particle moves in an ellipse under a force which is always directed towards its focus; find the law of force and the velocity at any point of its path. 5
- (b) A particle moves under a central repulsive force $\left\{ = \frac{m\mu}{(\text{distance})^3} \right\}$ and is projected from an apse at a distance a with velocity V . Show that the equation

26D/1841

(Turn Over)

to the path is $r \cos p\theta = a$, and that the angle θ describe in time t is

$$\frac{1}{p} \tan^{-1} \left(\frac{pV}{a} t \right)$$

where

$$p^2 = \frac{\mu + a^2 V^2}{a^2 V^2}$$

8

- (c) Show that the velocity of a particle moving in an ellipse about a centre of force in the focus is composed of two constant velocities, $\frac{\mu}{h}$ perpendicular to the radius and $\frac{\mu e}{h}$ perpendicular to the major axis.

6

2. (a) Find the law of force towards the pole which the curve $r^n = a^n \cos n\theta$ can be described.

5

- (b) A particle moves under a force $m\mu\{3au^4 - 2(a^2 - b^2)u^5\}$, $a > b$ and is being projected from an apse at a distance $a + b$ with velocity $\frac{\sqrt{\mu}}{(a + b)}$, show that the equation of the path is $r = a + b \cos \theta$.

7

(c) A particle moves with a central

acceleration $\left\{ = \frac{\mu}{(\text{distance})^2} \right\}$. It is

projected with velocity V at a distance R , show that its path is a rectangular hyperbola of the angle of projection is

$$\sin^{-1} \frac{\mu}{VR \left(V^2 - \frac{2\mu}{R} \right)^{1/2}} \quad 7$$

UNIT—II

3. (a) Find the moment of inertia of any triangular area ABC about a perpendicular to its plane through A . 7

(b) Show that the equation of the momental ellipsoid at the corner of a cube, of side $2a$, referred to its principal axes is

$$2x^2 + 11(y^2 + z^2) = \text{constant} \quad 6$$

(c) If A and B be the moments of inertia of a uniform lamina about two perpendicular axes OX and OY lying in its plane and F be the product of inertia of the lamina about these lines, find the moment of inertia and product of inertia about any two other axes OX', OY' inclined at an angle θ to OX, OY . 6

4. (a) Find the moment of inertia of an isosceles triangle about a perpendicular from the vertex upon the opposite side. 6
- (b) Find the moment of inertia of a right circular cylinder about its axis. 6
- (c) State and prove the theorem of perpendicular axes. 1+3=4
- (d) Find the moment of inertia of a uniform cube of side $2a$ about an axis through its centre and parallel to one face of the cube. 3

UNIT—III

5. (a) State and prove D'Alembert's principle on motion of a rigid body. 6
- (b) A uniform rod OA , of length $2a$, free to turn about its end O , revolves with uniform angular velocity ω about the vertical OZ through O , and is inclined at a constant angle α to OZ ; find the value of α . 6
- (c) A uniform rod, of mass m and length $2a$, can turn freely about one end which is fixed; it is started with angular velocity ω from the position in which it hangs vertically, find the motion. 7

6. (a) Deduce the general equations of motion of a rigid body in motion from D'Alembert's principle. 6

- (b) A plank, of mass M , is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man of mass M' , starting from the upper end walks down the plank so that it does not move; show that he gets to the other end in time

$$\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$$

where a is the length of the plank. 7

- (c) A fine string has two masses M and M' , tied to its ends and passes over a pulley, of mass m , whose centre is fixed; if the string does not slip over the pulley, show that M will descent with acceleration

$$\left(\frac{M - M'}{M + M' + m \frac{k^2}{a^2}} \right) g$$

where a is the radius and k the radius of gyration of the pulley. 6

UNIT—IV

7. (a) Find the equations of motion of a rigid body moving in two dimensions. 5
- (b) A uniform solid cylinder is placed with its axis horizontal on a plane, whose inclination to the horizon is α . Show that the least coefficient of friction between it and the plane, so that it may roll and not slide, is $\frac{1}{3}\tan\alpha$. 6
- (c) A uniform rod is held in a vertical position with one end resting upon a perfectly rough table, and when released rotates about the end in contact with the table. Find the normal reaction and friction when the rod is inclined at an angle θ to the vertical. 7
8. (a) Find the moment of momentum about the origin O of a rigid body M moving in two dimensions. 5
- (b) A uniform sphere rolls down an inclined plane, rough enough to prevent any sliding, show that the centre of sphere moves with constant acceleration
- $$\frac{a^2}{a^2 + k^2} g \sin\alpha$$
- where a is radius of sphere, k is radius of gyration and α is the angle of inclined plane with horizontal plane. 6

- (c) A rough uniform rod, of length $2a$, is placed on a rough table at right angles to its edge; if its centre of gravity be initially at a distance b beyond the edge, show that the rod will begin to slide when it has turned through an angle

$$\tan^{-1} \left(\frac{\mu a^2}{a^2 + 9b^2} \right)$$

where μ is the coefficient of friction. 7

★ ★ ★