

5M/MTH-302 Syllabus-2023

2 0 2 5

(Nov-Dec)

FYUP : 5th Semester Examination

MAJOR

MATHEMATICS

**(Numerical Methods and Optimization
Techniques)**

MTH-302

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

Answer **four** questions, selecting **one** from
each Unit

UNIT—I

1. (a) Compute the smallest positive root of
 $x^3 - 9x + 1 = 0$ by Bisection method,
correct to two decimal places. 6
- (b) Find a real root of the equation
 $x^4 - x - 10 = 0$, correct to three decimal
places by Newton-Raphson method. 5

(2)

- (c) Solve the following system by Gauss' elimination method :

7

$$\begin{aligned}2x + y + z &= 10 \\3x + 2y + 3z &= 18 \\x + 4y + 9z &= 16\end{aligned}$$

2. (a) Applying Newton's forward interpolation formula, find the cubic polynomial which takes the following values :

$$y(0) = 1, y(1) = 0, y(2) = 1, y(3) = 10$$

Hence or otherwise, obtain $y(4)$. $5+1=6$

- (b) Using Lagrange's interpolation formula, find the polynomial of degree two which attains the following tabular values :

6

x	1	2	4
y	-9	-12	-24

- (c) Find the polynomial of the lowest possible degree by applying Newton's divided difference formula from the following table :

6

x	3	2	1	-1
y	3	12	15	-21

(3)

UNIT—II

3. (a) Find $\frac{dy}{dx}$ at $x = 3.0$ for the function $y = f(x)$ from the following table : 6

x	3.0	4.0	5.0	6.0
y	4.0552	4.9530	6.0496	7.3891

- (b) Find the value of the integral

$$\int_0^1 \frac{dx}{1+x^2}$$

by using Simpson's $\frac{1}{3}$ rd rule. Hence, obtain the approximate value of π . $6+1=7$

- (c) Find the approximate value of

$$\int_0^1 \frac{x dx}{1+x^2}$$

up to three decimal places by using Trapezoidal rule, taking 6 equal subintervals of $[0, 1]$. 6

4. (a) Apply Euler's method to the initial value problem

$$\frac{dy}{dx} = x + y, \quad y = 0$$

when $x = 0$ from $x = 0$ to $x = 1.0$, taking $h = 0.2$. 7

(4)

- (b) Using Runge-Kutta method of fourth order, solve

$$\frac{dy}{dx} = x + y^2 \text{ with } y(0) = 1$$

at $x = 0.2$. Take $h = 0.2$.

6

- (c) Using Runge-Kutta method of order two, compute $y(0.2)$ for the equation

$$\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$$

Take $h = 0.2$.

6

UNIT—III

5. (a) Explain the following : 3+1=4

(i) The characteristics of the canonical form of LPP

(ii) Slack and surplus variables

- (b) Rewrite the following LPP in standard form : 5

$$\text{Minimize } Z = 2x_1 + x_2 + 4x_3$$

subject to the constraints

$$-2x_1 + 4x_2 \leq 4$$

$$x_1 + 2x_2 + x_3 \geq 5$$

$$2x_1 + 3x_3 \leq 2$$

$$x_1, x_2 \geq 0$$

and x_3 unrestricted in sign.

(5)

- (c) Show that the following system of linear equations has a degenerate solution : 5

$$2x_1 + x_2 - x_3 = 2$$

$$3x_1 + 2x_2 + x_3 = 3$$

- (d) Find all the basic feasible solutions of the following LPP : 5

$$\text{Maximize } Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$$

subject to the constraints

$$2x_1 + 3x_2 + x_3 + 4x_4 = 8$$

$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$x_1, x_2, x_3, x_4 \geq 0$$

and choose the one which maximizes Z .

6. (a) Use simplex method to solve the following LPP : 13

$$\text{Maximize } Z = 2x_1 + 3x_2$$

subject to the constraints

$$x_1 + x_2 \leq 4$$

$$-x_1 + x_2 \leq 1$$

$$x_1 + 2x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

- (b) Obtain the dual problem of the following LPP : 6

$$\text{Maximize } Z = 2x_1 + 5x_2 + 6x_3$$

(6)

subject to the constraints

$$5x_1 + 6x_2 - x_3 \leq 3$$

$$-2x_1 + x_2 + 4x_3 \leq 4$$

$$x_1 - 5x_2 + 3x_3 \leq 1$$

$$-3x_1 - 3x_2 + 7x_3 \leq 6$$

$$x_1, x_2, x_3 \geq 0$$

UNIT—IV

7. (a) Prove that the necessary and sufficient condition for the existence of a feasible solution to the general transportation problem is that

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j = \lambda$$

where a_i = quantity of commodity available at origin i , b_j = quantity of commodity needed at destination j . 5

- (b) Determine an initial basic feasible solution of the following transportation problem using Vogel's approximation method : 10

Source	Destination				Availability
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Requirement	60	40	30	110	240

(7)

(c) Explain the following : $2+2=4$

(i) Duality in transportation problem

(ii) Loops in transportation table

8. (a) With reference to a transportation problem, explain the following : $2+3=5$

(i) Optimality test

(ii) Degeneracy

(b) Solve the following transportation problem : 14

Source	Destination				Availability
	1	2	3	4	
1	21	16	25	13	11
2	17	18	14	23	13
3	32	27	18	41	19
Requirement	6	10	12	15	43

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