

4/MTH-250 Syllabus-2023

2026

(May-June)

FYUP : 4th Semester Examination

MATHEMATICS

(Calculus—II)

(MTH-250)

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks
for the questions.*

Answer **four** questions, selecting **one** from each Unit.

UNIT—I

1. (a) If f is a real-valued function defined on $[0, 1]$ by $f(x) = x \forall x \in [0, 1]$, then prove that f is Riemann integrable and

$$\int_0^1 f(x) dx = \frac{1}{2}$$

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(Turn Over)

(b) Show that if a real-valued function f is monotonic in $[a, b]$, then it is integrable in $[a, b]$. 4

(c) If f and g are two real-valued functions both bounded and integrable in $[a, b]$ then $f + g$ is also bounded and integrable in $[a, b]$ and

$$\int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx \quad 3+3=6$$

(d) Show with the help of an example that the integrability of $|f|$ may not imply the integrability of the real-valued function f . 3

2. (a) If real-valued function f is bounded and integrable on $[a, b]$ and m and M be the respective lower and upper bounds of f , then show that

$$m(b-a) \leq \int_a^b f(x) dx \leq M(b-a) \quad 5$$

(b) Let f be a continuous function. Show that the integral function $\phi(t) = \int_a^t f(x) dx$ is differentiable and $\phi' = f$. 4

(c) State and prove Weierstrass' second mean value theorem. 1+5=6

- (d) If a real-valued function f is bounded and integrable on $[a, b]$ and $f(x) \geq 0 \forall x \in [a, b]$, then show that

$$\int_a^b f(x) dx \geq 0. \quad 4$$

UNIT—II

3. (a) Show that if $f_n(x) = nx e^{-nx^2}$, the sequence $\{f_n\}$ is pointwise but not uniformly convergent in $[0, \infty[$. 5

- (b) Test the convergence of

$$\int_0^1 \frac{\sin\left(\frac{1}{x}\right)}{\sqrt{x}} dx \quad 4$$

- (c) Show that $\int_0^\infty x^{n-1} e^{-x} dx$ is convergent if and only if $n > 0$. 6

- (d) If $\{f_n\}$ is a sequence of continuous functions on an interval $[a, b]$, and if $f_n \rightarrow f$ uniformly on $[a, b]$, then f is continuous on $[a, b]$. Justify this statement. 4

4. (a) Examine the convergence of

$$\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/2}} \quad 5$$

(b) State and prove Abel's test for uniform convergence of a series of functions. 1+5=6

(c) Prove that $\sum a_n n^{-x}$, is uniformly convergent on $[0, 1]$ if $\sum a_n$ converges uniformly in $[0, 1]$. 4

(d) Discuss the convergence of $\int_0^{\infty} e^{-ax} \cos x dx$. 4

UNIT—III

5. (a) Find the value of

$$\int_C (x + y^2) dx + (x^2 - y) dy$$

taken in the clockwise sense along the closed curve C formed by $y^2 = x$ and $y = x$ between $(0, 0)$ $(1, 1)$. 5

(b) Express the double integral $\iint f(x, y) dx dy$ as a repeated integral, where the domain of integration is $\{(x, y) / x^2 + y^2 \leq a^2\}$. 4

(c) Change the order of integration in

$$\int_0^{\infty} \left\{ \int_x^{\infty} \frac{e^{-y}}{y} dx \right\} dy$$

and hence find the value. 4

(d) Evaluate

$$\iint \frac{\sqrt{a^2b^2 - b^2x^2 - a^2y^2}}{\sqrt{a^2b^2 + b^2x^2 + a^2y^2}} dx dy$$

The field of integration being the positive quadrant of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad 6$$

6. (a) Find the volume bounded by $y^2 + z^2 = 4ax$, $y^2 = ax$, $x = 3a$. 6

(b) Evaluate

$$\iiint (1-x-y-z)^{l-1} x^{m-1} y^{n-1} z^{p-1} dx dy dz$$

over the interior of the tetrahedron bounded by the planes $x=0$, $y=0$, $z=0$, $x+y+z=1$. 7

(c) Express the iterated integral

$$\int_0^1 \int_0^{x^2} \int_0^y f(x, y, z) dz dy dx$$

as a triple integral and then rewrite it as an iterated integral in a different order integrating first with respect to x , then z and then y . 6

UNIT—IV

7. (a) Prove that $\int \frac{x^2 + y^2}{p} ds$ taken round the

ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to

$$\frac{1}{4} \pi ab \{4 + (a^2 + b^2)(a^{-2} + b^{-2})\}$$

where p is the perpendicular from the centre to the tangent. 6

- (b) Evaluate the value of

$$\iiint_S p(x^4 + y^4 + z^4) dS$$

where S is the surface

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

and p is the perpendicular from the origin to the tangent plane at the point (x, y, z) . 6

- (c) State and prove Stokes' theorem. 1+5=6

8. (a) Verify Green's theorem by evaluating it in two ways the line integral

$$\int_C (x^3 + y^2) dx + (x^2 + y^3) dy$$

taken along the triangle whose vertices are $(0, 0)$, $(1, 0)$ and $(2, 1)$. 6

(7)

(b) Apply Stokes' theorem to evaluate

$$\int_C x^2 y^3 dx + dy + z dz$$

where C is the circle

$$x^2 + y^2 = a^2, \quad z = 0$$

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(c) Evaluate

$$\iint_S x dy dz + y dz dx + z dx dy$$

taken over the surface of the cube
[0, a; 0, a; 0, a].

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