

**3/MTH-200 Syllabus-2023**

**2 0 2 5**

( Nov-Dec )

**FYUP : 3rd Semester Examination**

**MAJOR**

**MATHEMATICS**

( Calculus-I and Statics )

**MTH-200**

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

Answer **one** question from each Unit

**UNIT—I**

1. (a) Show that a monotonic increasing sequence bounded above is convergent and it converges to its least upper bound.

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( Turn Over )

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(b) State whether the following statements are True or False with brief justification :  $2 \times 3 = 6$

(i) A bounded sequence is always convergent.

(ii) The sequence  $\left\{ \frac{4n+3}{n+2} \right\}$  converges to 4.

(iii) If  $\lim_{n \rightarrow \infty} u_n = 0$ , then the series  $\sum_{n=1}^{\infty} u_n$  is always convergent.

(c) State Cauchy's general principle of convergence of a sequence and apply it to show that the sequence  $\{x_n\}$  is divergent if

$$x_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \quad 1+4=5$$

(d) Is the sequence  $\{n^2\}$  a Cauchy sequence? Justify your answer. 2

2. (a) Test the convergence of the following series (any three) :  $2 \times 3 = 6$

(i)  $\frac{1}{1.2.3} + \frac{1}{3.4.5} + \frac{1}{4.5.6} + \dots \infty$

(ii)  $\frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \dots$

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(iii)  $\sum_{n=1}^{\infty} \frac{1}{n^n}$

(iv)  $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$

- (b) What is an alternating series? State Leibnitz's test for the convergence of an alternating series and show that the series

$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$  converges.  $1+1+4=6$

- (c) Show that the series

$$\sum \frac{\sqrt{n}x^n}{\sqrt{n^2 + 1}}, \quad x > 0$$

converges for  $x < 1$  and diverges for  $x \geq 1$ . 4

- (d) Test the absolute convergence of the

series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ .

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#### UNIT—II

3. (a) Find the points on the curve  $y = x^2 - 4x + 9$ , the tangents at which pass through the origin. 4

- (b) Find the radius of curvature at any point  $t$  on the curve

$x = a(\cos t + t \sin t), y = a(\sin t - t \cos t)$  3

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(c) Find the asymptotes of

$$x^3 + 2x^2y + xy^2 - x + 1 = 0 \quad 3$$

(d) Evaluate the following limits (any three) :  $3 \times 3 = 9$

(i)  $\text{Lt}_{x \rightarrow 1} \left\{ \frac{x}{x-1} - \frac{1}{\log x} \right\}$

(ii)  $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$

(iii)  $\lim_{x \rightarrow 0} (\cos x)^{\frac{1}{x^2}}$

(iv)  $\lim_{x \rightarrow 1} \frac{\log(1-x)}{\cot(\pi x)}$

(v)  $\text{Lt}_{x \rightarrow \infty} \frac{x^4}{e^x}$

4. (a) State and prove Taylor's theorem in Cauchy's form of remainder. 7

(b) Expand  $\sin x$  in a finite series in power of  $x$  with remainder in Cauchy's form. 4

(c) Apply Maclaurin's theorem to  $f(x) = (1+x)^4$  to deduce that

$$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4 \quad 5$$

(d) Find the pedal equation of the curve  $r = e^\theta$ . 3

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UNIT—III

5. (a) Two forces  $P$  and  $Q$  acting on a particle at an angle  $\alpha$  have a resultant  $(2k+1)\sqrt{P^2+Q^2}$ . When they act at an angle  $90^\circ-\alpha$ , the resultant becomes  $(2k-1)\sqrt{P^2+Q^2}$ . Prove that

$$\tan \alpha = \frac{k-1}{k+1} \quad 5$$

- (b) State and prove Lami's theorem.  $1+4=5$

- (c) If the two like-parallel forces  $P$  and  $Q$  acting on a rigid body at  $A$  and  $B$  be interchanged in position, then show that the point of application of the resultant will be displaced along  $\overrightarrow{AB}$  through a distance  $d$  where

$$d = \frac{P-Q}{P+Q} AB \quad (P > Q) \quad 5$$

- (d) Three forces  $P$ ,  $Q$ ,  $R$  acting at the vertices  $A$ ,  $B$ ,  $C$  respectively of a triangle, each perpendicular to the opposite side, keep it in equilibrium. Prove that  $P:Q:R = a:b:c$ .  $4$

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6. (a) Three forces acting along the sides of a triangle, taken in order, are equivalent to a couple. Show that their magnitudes are proportional to the sides of the triangle. 5
- (b) Three forces  $P$ ,  $Q$ ,  $R$  act in the same sense along the sides  $\overrightarrow{BC}$ ,  $\overrightarrow{CA}$ ,  $\overrightarrow{AB}$  of a triangle  $ABC$ . Show that, if their resultant passes through the in-centre, then  $P + Q + R = 0$ . 4
- (c) If the resultant of two equal forces inclined at an angle  $2\theta$  is twice as great as when they are inclined at an angle  $2\phi$ , then prove that  $\cos\theta = 2\cos\phi$ . 5
- (d) Forces of magnitudes 1, 2, 3, 4,  $2\sqrt{2}$  act respectively along the sides  $\overrightarrow{AB}$ ,  $\overrightarrow{BC}$ ,  $\overrightarrow{CD}$ ,  $\overrightarrow{DA}$  and the diagonal  $\overrightarrow{AC}$  of the square  $ABCD$ . Show that their resultant is a couple, and find its moment. 5

UNIT—IV

7. (a) Forces proportional to 1, 2, 3, 4 act along the sides  $\overrightarrow{AB}$ ,  $\overrightarrow{BC}$ ,  $\overrightarrow{AD}$ ,  $\overrightarrow{DC}$  respectively of a square  $ABCD$ , the length of whose sides is 2 m. Find the magnitude and line of action of the resultant. 5

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(b) A heavy uniform rod of length  $2a$  rests in equilibrium, having one end against a smooth vertical wall, and being placed upon a peg at a distance  $b$  from the wall. Show that the inclination of the rod to the horizontal is  $\cos^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$ .

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(c) A uniform ladder is in equilibrium with one end resting on the ground and the other against a vertical wall; if the ground and wall be both rough, the coefficients of friction being  $\mu$  and  $\mu'$  respectively, and if the ladder be on the point of slipping on both ends, then show that the inclination of the ladder to the horizon is given by  $\tan \theta = \frac{1 - \mu\mu'}{2\mu}$ .

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(d) Find the centre of gravity of a parallelogram formed by four uniform rods.

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8. (a) The moments of a system of coplanar forces acting in the  $(x, y)$ -plane about  $(0, 0)$ ,  $(a, 0)$ ,  $(0, a)$  are  $aw$ ,  $2aw$ ,  $3aw$  respectively. Find the magnitudes of the components parallel to the coordinate axes and the line of action of the single force to which the system is equivalent.

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- (b) The least force which will move a weight up an inclined plane is  $P$ . Show that the least force, acting parallel to the plane, which will move the weight upwards is  $P\sqrt{1+\mu^2}$ , where  $\mu$  is the coefficient of friction.

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- (c) Forces  $P, Q, R, S$  act along the sides  $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DA}$  of the cyclic quadrilateral  $ABCD$ , taken in order, where  $A$  and  $B$  are the extremities of a diameter. If they are in equilibrium, then prove that

$$R^2 = P^2 + Q^2 + S^2 + \frac{2PQS}{R}$$

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- (d) A uniform rod  $AB$  is suspended with its end in contact with a smooth vertical wall  $AC$  by a string  $CE$ . Show that  $CB$  is horizontal if  $AE = \frac{1}{3}AB$ .

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