

3/MTH-200 Syllabus-2023

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(December)

FYUP : 3rd Semester Examination

MAJOR

MATHEMATICS

(Calculus—I and Statics)

MTH-200

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

Answer **four** questions, choosing **one** from each Unit

UNIT—I

1. (a) Show that a convergent sequence is bounded. Is the converse true? Justify your answer. 3+2=5

- (b) Show that the sequence $\{a_n\}$, where

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$$

is monotonic increasing. Also, show that it is bounded. Is it convergent? 3+3+1=7

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(Turn Over)

- (c) Evaluate the limits of the following sequences as $n \rightarrow \infty$ (any two) : $2 \times 2 = 4$

(i) $\frac{6n^3 + 2n + 1}{n^3 + n^2}$

(ii) $\sqrt[3]{n+1} - \sqrt[3]{n}$

(iii) $\sqrt{n^2 + n} - \sqrt{n}$

- (d) Give an example of a convergent series which is not absolutely convergent. 2

2. (a) Test the convergence of the following series (any three) : $2 \times 3 = 6$

(i) $\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots$

(ii) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1} \right)^n$

(iii) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(iv) $\sum_{n=1}^{\infty} (\sqrt{n^2 + 1} - n)$

(v) $\frac{1}{2 \log 2} + \frac{1}{3 \log 3} + \dots$

- (b) Evaluate the following limit : 5

$$\text{Lt}_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \dots + \frac{1}{\sqrt{n^2 + n}}$$

- (c) Determine whether the series

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$$

is convergent.

3

- (d) (i) Test the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n^n}$$

for absolute convergence.

2

- (ii) For what values of x is the series

$$\frac{x}{1^2} + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \dots$$

convergent?

2

UNIT—II

3. (a) Find the equation of the tangent and normal to the curve

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$$

at the point (x', y') .

2+2=4

- (b) Find the radius of curvature of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at the point (x, y) .

3

- (c) Find the asymptotes of

$$x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 = 2$$
 3

- (d) Evaluate the following limits (any three) :

$$3 \times 3 = 9$$

(i) $\lim_{x \rightarrow 0} x^{2 \sin x}$

(ii) $\lim_{x \rightarrow \infty} \frac{\log x}{x^a}, (a > 0)$

(iii) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{1 - \cot x}$

(iv) $\lim_{x \rightarrow 1} x^{\frac{1}{1-x}}$

(v) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$

4. (a) State and prove Taylor's theorem in Cauchy's form of remainder. 7
- (b) Expand $\log(1+x)$ in a finite series in powers of x with remainder in Cauchy's form. 4
- (c) Find the points on the curve

$$y = x^2 - 4x + 9$$

 the tangents at which pass through the origin. 3
- (d) Expand $\sin x$ in Maclaurin's infinite series. 5

UNIT—III

5. (a) The resultant of two forces P and $2P$ acting at a point is perpendicular to P . Find the angle between the forces. 5
- (b) State and prove Varignon's theorem. 5
- (c) P and Q are like parallel forces. If P is moved parallel to itself through a distance x , then show that the resultant of P and Q moves through a distance $\frac{Px}{P+Q}$ 5
- (d) Show that the forces 3 kg, 8 kg, 7 kg, 11 kg and 5 kg weight acting respectively along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ and $\overrightarrow{AC}$ of a rectangle $ABCD$ are equivalent to a couple if $AB = 4$ m and $BC = 3$ m. Show that the moment of the couple is 53 kg-metre. 4
6. (a) State and prove Lami's theorem. 5
- (b) ABC is a triangle and D , E , F are the middle points of the sides BC , CA , AB respectively. Show that the forces acting on a particle and represented by $\overrightarrow{AD}$, $\overrightarrow{BE}$ and $\overrightarrow{CF}$ will maintain equilibrium. 4

- (c) Two forces P and Q acting at a point have got a resultant R . If Q be doubled, R is also doubled. Again, if Q be reversed in direction, then also R is doubled. Show that

$$P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2} \quad 5$$

- (d) Three forces P, Q, R act in the same sense along the sides $\overrightarrow{BC}, \overrightarrow{CA}, \overrightarrow{AB}$ of a triangle ABC . Show that if their resultant passes through the circum-centre, then

$$P \cos A + Q \cos B + R \cos C = 0 \quad 5$$

UNIT—IV

7. (a) Show that a system of coplanar forces can be reduced to two forces, one of which acts through a given point and the other along a given straight line. 4
- (b) A uniform ladder rests in limiting equilibrium with the lower end on a rough horizontal plane and its upper end against a smooth vertical wall. If θ be the inclination of the ladder to the vertical, then prove that $\tan \theta = 2\mu$. 5

- (c) A heavy uniform rod of length a rests with one end against a smooth vertical wall, the other end being tied to a point of the wall by a string l . Prove that the rod may remain in equilibrium at an angle θ to the wall, given by

$$\cos^2 \theta = \frac{l^2 - a^2}{3a^2} \quad 5$$

- (d) Find the CG of a uniform triangular lamina. 5

8. (a) A body of mass 100 kg is placed on a rough inclined plane and is just supported by a force of magnitude 40 kg (force) applied along the plane. When the force is increased to 80 kg (force), the body is on the point of sliding up. Find the coefficient of friction. 5

- (b) A heavy uniform rod can turn freely about one of its ends and is pulled aside from the vertical by a horizontal force acting at the other end of the rod, equal to half its weight. Prove that the rod will rest at an inclination of 45° to the vertical. 5

(c) The forces 1 kg, 2 kg, 4 kg, 5 kg act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ of a square $ABCD$ and a force P acts at the centre of the square. If the five forces are equivalent to a couple, then find the magnitude and direction of P .

4

(d) D, E, F are the midpoints of the sides $\overrightarrow{BC}$, $\overrightarrow{CA}$, $\overrightarrow{AB}$ of the triangle ABC . Masses m_1, m_2, m_3 are placed at A, B, C and masses μ_1, μ_2, μ_3 are placed at D, E, F . If the two systems have the same CG, then prove that

$$\frac{m_1}{\mu_2 + \mu_3} = \frac{m_2}{\mu_3 + \mu_1} = \frac{m_3}{\mu_1 + \mu_2}$$

5
