

- (b) Prove that the equation

$$2x^2 - 5xy + 3y^2 - 2x + 3y = 0$$

represents two lines and find their point of intersection.

5

- (c) Through what angle must the axes be rotated so that the term of xy may be removed from the equation

$$ax^2 + 2hxy + by^2 = 0 ?$$

4

- (d) Find the diameter of the conic

$$4x^2 + 6xy - 5y^2 = 1$$

conjugate to the diameter $y = 2x$.

4

2. (a) If the two pairs of lines

$$x^2 - 2pxy - y^2 = 0 \text{ and } x^2 - 2qxy - y^2 = 0$$

be such that each pair bisects the angles between the other pair, prove that $pq + 1 = 0$.

5

- (b) Reduce the equation

$$3x^2 - 6xy - 5y^2 - 6x + 22y - 17 = 0$$

to standard form.

7

- (c) Find the equation of the tangent to the conic

$$4x^2 + 3xy + 2y^2 - 3x + 5y + 7 = 0$$

at the point $(1, -2)$.

3

(d) Determine the conic-section represented by the following equations : $1\frac{1}{2} + 1\frac{1}{2} = 3$

(i) $4x^2 - 20xy + 25y^2 - 7x + 11y - 8 = 0$

(ii) $7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$

UNIT—II

3. (a) Find the equation of a cone whose vertex is $(1, -2, 1)$ and whose guiding curve is $z = 0, 2x^2 + y^2 = 1$. 5

(b) Find the equations of the two tangent planes to the sphere $x^2 + y^2 + z^2 = 9$ which passes through the line $\{x + y = 6, x - 2z = 3\}$. 5

(c) Find the equation of the plane passing through the intersection of the planes $2x + y - z = 9$ and $7x - 3y + 4z = 2$ and through the point $(3, -1, 2)$. 5

(d) Find the centres and radii of the following spheres : $2+2=4$

(i) $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$

(ii) $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$

4. (a) Find the equation of a plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the plane $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$. 6
- (b) Obtain the equation of a sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as a great circle. 5
- (c) Find the enveloping cone of the sphere $x^2 + y^2 + z^2 - 2x + 4z = 1$ with its vertex at $(1, 1, 1)$. 5
- (d) Find the angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$. 3

UNIT—III

5. (a) Find the domain and range of $f(x, y) = \frac{1}{\sqrt{x+y-1}}$ 4
- (b) Determine whether the function f defined by $f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2}, & \text{where } x^2 + y^2 \neq 0 \\ 0, & \text{where } x = y = 0 \end{cases}$ is continuous at $(0, 0)$. 5

(c) If $f(x, y) = x \cos y + ye^x$, then find

$$\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial y^2} \text{ and } \frac{\partial^2 f}{\partial x \partial y}. \quad 5$$

(d) If

$$u = \sin^{-1} \frac{5xy + 3x^2}{\sqrt{4x + 5y}}$$

show that

$$3 \left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \right) = 5 \tan u \quad 5$$

6. (a) Verify Euler's theorem for

$$f(x, y, z) = axy + byz + czx \quad 4$$

(b) Let $f(x, y)$ be a real valued function defined by

$$f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & x^2 + y^2 \neq 0 \\ 0, & x^2 + y^2 = 0 \end{cases}$$

Evaluate $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ using definition. 5

(c) Show that the double limit exists but repeated limits do not exist of the function

$$f(x, y) = \begin{cases} x \sin \frac{1}{y} + y \sin \frac{1}{x}, & \text{when } x, y \neq 0 \\ 0, & \text{when } x = y = 0 \end{cases} \quad 5$$

- (d) Find the degree of the homogeneous function

$$f(x, y) = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$$

and show that

$$2x \frac{\partial f}{\partial x} + 2y \frac{\partial f}{\partial y} = f(x, y) \quad 5$$

UNIT—IV

7. (a) Prove that

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix} \quad 5$$

- (b) If $\vec{u} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{v} = \sin t\hat{i} - \cos t\hat{j}$, then find

$$(i) \frac{d}{dt}(\vec{u} \cdot \vec{v})$$

$$(ii) \frac{d}{dt}(\vec{u} \cdot \vec{u}) \quad 3+2=5$$

- (c) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ , where Q has coordinates $(5, 0, 4)$. 5

- (d) Verify that $\text{curl grad } f = \vec{0}$, where $f = x^2y + 2xy + z^2$. 4

8. (a) Show that

$$\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$$

where $r = |\vec{r}|$.

5

- (b) Find the unit vector normal to the surface $\phi = 2x^2y + 3yz - 4$ at the point $(1, -1, -2)$.

5

- (c) Determine the divergence and curl of the vector

$$\vec{f} = (x^2 - y^2)\hat{i} + 2xy\hat{j} + (y^2 - xy)\hat{k}$$

5

- (d) If $\vec{a} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$, and

$$\vec{b} = (2t-3)\hat{i} + \hat{j} - t\hat{k}, \quad \text{then find}$$

$$\frac{d}{dt}\left(\vec{a} \times \frac{d\vec{b}}{dt}\right) \text{ at } t = 2.$$

4

2m/PHY-150(Th) Syllabus-2023

2026

(May-June)

FYUP : 2nd Semester Examination

PHYSICS

(Minor)

**(Electricity and Magnetism, Optics
and Electronics)**

(PHY-150)

(Theory)

Marks : 56

Time : 3 hours

*The figures in the margin indicate full marks
for the questions.*

Answer any **eight** questions.

1. (a) Obtain the differential form of Gauss' law in electrostatics. 3
- (b) Using Gauss' law, calculate the electric field due to a uniformly charged rod. 4

26D/1605

(Turn Over)