

1M/MTH-100 Syllabus-2023

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(Nov-Dec)

FYUP : 1st Semester Examination

MAJOR

MATHEMATICS

(Fundamental Mathematics—I)

MTH-100

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

Answer **one** question from each Unit

UNIT—I

1. (a) Use ϵ - δ definition to show that

$$\lim_{x \rightarrow 2} \frac{2x^2 - 8}{x - 2} = 8 \quad 4$$

- (b) A function $f(x)$ is defined as follows :

$$\begin{aligned} f(x) &= 3 + 2x ; \text{ for } -\frac{3}{2} \leq x < 0 \\ &= 3 - 2x ; \text{ for } 0 \leq x < \frac{3}{2} \\ &= -3 - 2x ; \text{ for } x \geq \frac{3}{2} \end{aligned}$$

Show that $f(x)$ is continuous at $x = 0$
and discontinuous at $x = \frac{3}{2}$. 4

(2)

(c) Evaluate :

$$3 \times 2 = 6$$

(i) $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x})$

(ii) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$

(d) State the intermediate value theorem.

Use the theorem to show that the equation $\cos x = x$ has at least one solution.

$$1 + 3 = 4$$

2. (a) Find $f(0)$ so that $f(x) = \frac{1 - \cos x}{x^2}$ for $x \neq 0$

may be continuous at $x = 0$.

4

(b) Use ϵ - δ definition to show that $f(x)$ is continuous at $x = 0$

$$f(x) = x \sin\left(\frac{1}{x}\right), \text{ when } x \neq 0$$

$$= 0, \text{ when } x = 0$$

Find δ if $\epsilon = 0.1$.

$$3 + 1 = 4$$

(c) Show that the limit

$$\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$$

does not exist.

3

(d) Evaluate :

3

$$\lim_{x \rightarrow \infty} \frac{x + \sin x}{x + \cos x}$$

(3)

- (e) State and prove fixed-point theorem.

1+3=4

UNIT—II

3. (a) Differentiate $\sin x$ w.r.t. x^2 . 2

- (b) If $y = (\sin^{-1} x)^2$, prove that—

(i) $(1 - x^2)y_2 - xy_1 = 2$

(ii) $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$

2+3=5

- (c) Find the range of values for which the function $f(x) = x^3 - 6x^2 - 36x + 7$ increases with x . 3

- (d) Show that the curve $y^3 = 8x^2$ is concave to the foot of the ordinate everywhere except at the origin. 3

- (e) Discuss the applicability of the mean value theorem $f(b) - f(a) = (b - a)f(\xi)$, $a < \xi < b$. Find ξ , if the theorem be applicable for the function

$f(x) = x(x-1)(x-3); 0 \leq x \leq 4$ 2+4=6

4. (a) If a function has a finite derivative at a point, prove that it is continuous at that point. Show also by an example that the converse is not necessarily true. 3+2=5

(4)

(b) Find the n th derivative of

$$y = \frac{1}{ax+b} \quad 3$$

(c) Find the points of inflexion of the curve

$$y = e^{-x^2} \quad 2$$

(d) Show that

$$\frac{x}{1+x} < \log(1+x) < x \quad \text{for } x > 0 \quad 5$$

(e) Use differentials to find approximate value of $\cos 61^\circ$. 4

UNIT—III

5. (a) Evaluate by the method of summation,

$$\int_1^2 (3x^2 + 2) \text{ and verify the result with the help of fundamental theorem.} \quad 4+1=5$$

(b) Evaluate : 3 \times 2 = 6

$$(i) \int \frac{2x+1}{x^2+2x+5} dx$$

$$(ii) \int \frac{dx}{(1-x)\sqrt{1-x^2}}$$

- (c) Obtain a reduction formula for

$$\int_0^{\frac{\pi}{4}} \tan^n x \, dx$$

n being a positive integer greater than 1. 4

- (d) Find the area above the x -axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$. 4

6. (a) Prove that

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} \, dx = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1) \quad 5$$

- (b) If $f(x) = |x - 2|$, show that $\int_0^3 f(x) \, dx = \frac{5}{2}$. 3

- (c) Evaluate : 3

$$\int \sqrt{\frac{x-1}{x+1}} \, dx$$

- (d) Find the length of the arc of the parabola $x^2 = 4y$ from the vertex to the point where $x = 2$. 4

- (e) Find the volume of the solid formed by rotation of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the major axis. 4

UNIT—IV

7. (a) If z_1 and z_2 are two complex numbers,

then show that $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$ and

$$\text{amp} \left(\frac{z_1}{z_2} \right) = \text{amp } z_1 - \text{amp } z_2. \quad 3+2=5$$

- (b) Solve the equation $x^3 - 12x + 65 = 0$ by Cardan's method. 5

- (c) Express $x^5 + 5x^3 + 3x$ as a polynomial in $(x-1)$. 4

- (d) Find the condition that the equation

$$x^4 + px^3 + qx^2 + rx + s = 0$$

should have the roots connected by the relation $\beta + \gamma = \alpha + \delta$. 5

8. (a) Prove that the equation $x^4 + 15x^2 + 7x - 11 = 0$ has two real roots, one positive and the other negative and two other complex roots. 4

- (b) Solve the equation

$$x^4 - 13x^3 + 53x^2 - 83x + 42 = 0$$

if two of the roots are 1 and 2. 5

- (c) (i) State De Moivre's theorem.

- (ii) Solve the equation

$$x^8 + x^5 - x^3 - 1 = 0 \quad 1+4=5$$

(7)

(d) Solve the equation

$$6x^3 - 11x^2 + 6x - 1 = 0$$

given that the roots are in AP.

5
