

1m/MTH-100 Syllabus-2023

2024

(December)

FYUP : 1st Semester Examination

MINOR

MATHEMATICS

(Fundamental Mathematics—I)

MTH-100

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

Answer **four** questions, selecting **one** from
each Unit

UNIT—I

1. (a) Use ϵ - δ definition of limits to prove that

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4 \quad 4$$

- (b) Evaluate the following limits : 3+3=6

(i) $\lim_{x \rightarrow \infty} \frac{x^3 - 5x^2 + 1}{7x^3 + 4x - 3}$

(ii) $\lim_{x \rightarrow -\infty} \frac{4x^2 - x}{2x^3 - 5}$

D25/880

(Turn Over)

(c) Let

$$f(x) = \begin{cases} 3x-1 & x < 0 \\ 5x^2+2 & x = 0 \\ 2-8x & x > 0 \end{cases}$$

Does $f(x)$ has a removable discontinuity at $x=0$? Justify your answer. 4

(d) Show that the function

$$f(x) = x^2 + \sin x - 1$$

has a real root between 0 and $\frac{\pi}{2}$. 4

2. (a) Show that $\lim_{x \rightarrow 0} \frac{|x|}{x}$ does not exist. 3

(b) Use ϵ - δ definition of continuity to prove that the following function is continuous at $x=0$: 5

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

(c) Let $f : [a, b] \mapsto \mathbb{R}$ be a function. Explain the continuity of f from a geometrical perspective. 2

(d) Evaluate the limit $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$. 3

(e) Prove that if a real-valued function f is continuous on $[a, b]$ and $f(x) \in [a, b]$, then f has a fixed point in $[a, b]$. 5

UNIT—II

3. (a) Let f be a real-valued function and $x_0 \in \mathbb{R}$. Define $f'(x_0)$. Use it to evaluate the derivative of $f(x) = \sqrt{x}$ at $x = x_0$. $1+3=4$
- (b) Let f be a real-valued function defined over some open interval of $\mathbb{R}$. Prove that if f is differentiable at a point, then f is continuous there. Is the converse true? Justify your answer. $3+2=5$
- (c) Show that $f(x) = \sin^2 x$ has a relative minimum at $x = 0$. 3
- (d) Verify Rolle's theorem for the function $f(x) = x^2 - 8x + 15$ defined over the closed interval $[3, 5]$. 3
- (e) State Lagrange's mean value theorem. Use it to prove that if $f'(x) = 0$ for every $x \in (a, b)$, then f is constant in $[a, b]$. $1+3=4$
4. (a) Find the intervals on which the function $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$ is increasing and the intervals on which it is decreasing. 3
- (b) Find the intervals on which the function $f(x) = x^4 - 5x^3 + 9x^2$ is concave up and concave down. Also, find all its inflection points. $2+2+2=6$

- (c) Let $f: [a, b] \mapsto \mathbb{R}$ be a function and $c \in [a, b]$. If $f''(c) = 0$, does it necessarily mean that c is a point of inflection? Justify your answer. 2
- (d) Find the absolute maximum and absolute minimum values of $f(x) = 2x^3 - 15x^2 + 36x$ on the interval $[1, 5]$. 2
- (e) Let $y = \tan^{-1} x$, show that—
 (i) $(1 + x^2)y_1 = 1$
 (ii) $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$ 2+4=6

UNIT—III

5. (a) If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then prove that $\int_a^b f(x)dx = F(b) - F(a)$. 7
- (b) Find the area under the curve $f(x) = 3\sqrt{x}$ over the interval $[1, 4]$. 2
- (c) Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$. 5
- (d) Find the arc length of the curve $y = x^{\frac{3}{2}}$ from $(1, 1)$ to $(2, 2\sqrt{2})$. 5

6. (a) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y -axis. 5
- (b) Find the volume of the solid that results when the region enclosed by the curves $y = e^x$, $y = 0$, $x = 0$, $x = \ln 3$ is revolved about the x -axis. 4
- (c) If $I_n = \int \sin^n x \, dx$, then prove that for n being a positive integer greater than 1

$$I_n = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} I_{n-2}$$

Deduce the reduction formula for

$$\int_0^{\frac{\pi}{2}} \sin^n x \, dx.$$

5+2=7

- (d) Using the reduction formula, obtain the value of $\int \tan^4 x \, dx$. 3

UNIT—IV

7. (a) If $f(x) = x^4 - x^3 + 2x^2 + 6x - 2$, find $f(x+2)$. 4
- (b) Solve the equation
- $$x^4 + 2x^3 + 2x^2 + 2x + 1 = 0$$
- given that it has multiple roots. 5

(c) Solve the equation

$$x^4 - x^3 + 2x^2 - 2x + 4 = 0$$

if one root is $1+i$.

5

(d) If α, β, γ are the roots of $x^3 + 5x - 6 = 0$,

find the value of $\sum \frac{\alpha^2}{\beta}$.

5

8. (a) Use Descartes rule of sign to determine the minimum number of complex roots of $f(x) = x^4 - x^3 + x^2 - x + 1$.

3

(b) Find the three values of $(1-i)^{\frac{1}{3}}$, using De Moivre's theorem.

5

(c) If α, β, γ be the roots of the equation $x^3 - px^2 + qx - r = 0$, form the equation whose roots are $\beta\gamma + \frac{1}{\alpha}, \gamma\alpha + \frac{1}{\beta}, \alpha\beta + \frac{1}{\gamma}$.

5

(d) Solve $x^3 - 9x + 28 = 0$ by Cardan's method.

6
