

4/ECO-251 Syllabus-2023

2026

(May-June)

FYUP : 4th Semester Examination

ECONOMICS

(Mathematical Methods for Economics II)

(ECO-251)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions.

Answer **five** questions, taking at least **one** from each Unit.

UNIT—I

1. (a) What is a vector space? What are the axioms that define a vector space? 2+2=4

- (b) For scalars $\alpha=3$, $\beta=4$ and vectors $u=(1, 2)$ and $v=(3, 4)$, verify the following property : 3

$$\alpha(u + v) = \alpha u + \alpha v$$

- (c) What are the conditions for non-singularity of a matrix? Test whether the following matrix is non-singular : 1+3=4

$$A = \begin{bmatrix} 7 & 9 & 5 \\ 3 & 0 & 1 \\ 10 & 8 & 6 \end{bmatrix}$$

- (d) If

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

prove that $|A \cdot B| = |A| \cdot |B|$.

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2. (a) Show without expansion that

$$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} c & b & a \\ b & a & c \\ a & c & b \end{bmatrix}$$

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- (b) If

$$\begin{bmatrix} x & 2 & -2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ 2 \\ -2 \end{bmatrix} = 0$$

find x .

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- (c) If

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 3 & 1 & -2 \\ -1 & 0 & 4 \end{bmatrix}$$

prove that $A^{-1} \cdot A = I$.

8

(3)

3. (a) Solve the following system of simultaneous equations using Cramer's rule :

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$$2x_1 + 3x_2 - x_3 = 15$$

$$4x_2 + 2x_3 = 16$$

$$3x_1 + 2x_2 = 18$$

- (b) Define rank of a matrix.

2

- (c) Find the rank of the matrix A

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 2 & 4 & -4 \end{bmatrix}$$

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UNIT—II

4. (a) What is a homothetic function? Are all homothetic functions homogeneous?

3+1=4

- (b) Show that the following functions are homogeneous of degree 1 :

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(i) $Q(L, K) = (K^{0.5} + L^{0.5})^2$

(ii) $P(L, K) = A[\alpha K^{-\rho} + (1 - \alpha)L^{-\rho}]^{-1/\rho}$

- (c) What is the condition for differentiability of a function at a point defined on an interval?

2

(d) Is the function

$$f(x) = 2 - x \quad \text{when } 1 \leq x \leq 2$$

$$= x - \frac{1}{2}x^2 \quad \text{when } x > 2$$

differentiable at $x=2$?

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5. (a) Find the maximum/minimum value of the function

$$u = 2x^2 + y^2 - 4x + 8y$$

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- (b) A multiproduct monopolist produces two commodities which are technically related in production. The joint cost for the firm is $C = q_1^2 + 2q_1q_2 + q_2^2$. The demand laws of the products are $p_1 = 10 - q_1$ and $p_2 = 20 - 2q_2$. Find the equilibrium values of q_1 , q_2 , p_1 , p_2 and the profit π .

6+3=9

6. (a) (i) What is the difference between 'constrained' and 'unconstrained' optimization?

- (ii) State the necessary and sufficient conditions for constrained optimization.

2+3=5

- (b) A firm can buy two inputs capital (K) and labour (L) at ₹ 18 per unit and ₹ 8 per unit respectively and faces the production function $Q = 24K^{0.6}L^{0.3}$. What is the maximum output it can produce for a budget of ₹ 50,000?

10

UNIT—III

7. (a) Distinguish between differential equation and difference equation. 3
- (b) Solve the following differential equations (any four) : $3 \times 4 = 12$
- (i) $3xydy = (x^2 + y^2)dx$
- (ii) $y(1+x)dx + x(1-y)dy = 0$
- (iii) $(2x+3y)dx + (3x-x^2y)dy = 0$
- (iv) $\frac{dy}{dx} + e^x + ye^x = 0$
- (v) $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$
8. (a) The demand and supply functions are given by $D_t = 5 - 3P_t$; $S_t = 6 + 8P_{t-1}$. Determine the time path of price if the initial price is ₹ 10. 6
- (b) Find the solutions of the following difference equations (any three) : $3 \times 3 = 9$
- (i) $y_t = Ly_{t+1}$, $y_0 = M$
- (ii) $y_{t+1} - 5y_t = 1$, $y_0 = \frac{7}{4}$
- (iii) $y_{x+1} = 0.2y_x + 4$, $y_0 = 4$
- (iv) $2y_{t+1} - y_t = 6$, $y_0 = 7$

9. (a) The marginal rate of substitution of Y for X is given by

$$\frac{-\alpha}{\beta} \cdot \frac{y+b}{x+a}$$

Show that the form of utility function would be $u = (x+a)^\alpha \cdot (y+b)^\beta$, where a , b , α , β are constants.

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- (b) The demand and supply functions in a market is given by

$$Q_d = a - bp$$

$$Q_s = -c + dp$$

Obtain the time path of price $P(t)$ if the rate of change of price over time is directly proportional to excess demand, the adjustment coefficient being $\lambda (\lambda > 0)$ and show that it is dynamically stable.

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UNIT—IV

10. (a) What are the assumptions of Leontief's static model?

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- (b) Point out the limitations of input-output analysis.

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(c) The following table gives the inter-industry transactions in millions of rupees for an economy with three sectors S_1, S_2, S_3 :

	S_1	S_2	S_3	Final demand	Total output
S_1	50	25	25	100	200
S_2	40	50	10	200	300
S_3	100	50	50	300	600

Calculate the coefficient matrix.

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11. (a) The technological coefficient matrix is given by

$$A = \begin{bmatrix} 0.3 & 0.2 & 0.4 \\ 0 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.2 \end{bmatrix}$$

Find the output level consistent with the model if $D_1=300$, $D_2=500$, $D_3=400$.

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- (b) What are the applications of linear programming techniques in the field of economics?

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12. (a) A manufacturing unit plans to produce two products I and II which will be stored in a storage area whose capacity is 30000 square feet. Product I takes 3 square feet of the space per unit, while product II requires 4 square feet per

unit. It takes 4 machine hours to manufacture a unit of product I and 8 machine hours are required for a unit of product II. During the production process, 48000 machine hours are available. Also 36000 man hours are available for finishing the products. A unit of product I takes 4 man hours for finishing and product II takes 3 man hours. Profit contributions are ₹ 20 per unit of product I and ₹ 60 per unit of product II.

In order to maximize profit, how many units of each product should be produced?

- (i) Formulate the linear programming problem.
 - (ii) Find the optimal solution graphically.
- (b) Why do we convert a primal problem in linear programming to a dual problem?
